Performance Analysis of a RTK Vector Phase-Locked Loop Architecture for GPS Signal Tracking in Degraded Environments

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Abstract
This paper investigates the application of a vector tracking approach to carrier phase tracking for improved tracking in degraded signal environments. In contrast to many vector-based carrier phase tracking approaches, the algorithms described in this paper do not rely on local loop filters to maintain phase lock. A relative position vector estimate, fixed-integer carrier ambiguities, and measurements from the base station are used to drive the carrier replica. To test carrier phase tracking approaches, Monte Carlo simulations were performed to compare the performance of a real-time kinematic (RTK) vector phase-locked loop (VPLL) approach with the performance obtained by scalar tracking. In these simulations, the RTK-VPLL maintains phase lock at carrier-to-noise density ratios 4 to 7 dB-Hz lower than the scalar tracking receiver. The proposed algorithms were tested using live sky data, and the RTK-VPLL performance was compared with the performance of a commercial global positioning system receiver. Significant reductions in carrier cycle slips and loss of lock were observed in moderate-foliage environments.

Keywords
degraded GNSS signals, real-time kinematic positioning, vector tracking

1 | INTRODUCTION

Civilian users have reaped the benefits of global positioning system (GPS) localization accuracy for years using hand-held devices capable of reporting the receiver position to within several meters of the true global position. These devices work well when there is a clear line of sight to the satellites transmitting the GPS signal; however, the signal strength received by the user is weak, approximately $10^{-16}$ W (Misra & Enge, 2011), and is therefore susceptible to interference. Signal disruptions are caused by a variety of environmental factors, including natural obstructions (e.g., trees, canyons), manmade physical obstructions (e.g., building, bridges), and electromagnetic interference (e.g., signal jammers). Currently, these factors, along with accuracy limitations, are the primary concerns that limit the adoption of GPS in markets such as the booming commercial autonomous vehicle industry.
To address these two concerns, accuracy and reliability, this paper focuses on the development and testing of improved signal tracking capabilities for reliable differential GPS (DGPS) positioning in degraded environments. DGPS is a technique that uses data from a static GPS receiver mounted at a known location to improve the localization accuracy of a mobile rover receiver. This technique relies on mitigating correlated errors experienced by both receivers, such as atmospheric interference and satellite clock errors. The receiver architecture developed in this paper combines DGPS positioning with a signal tracking approach known as vector tracking.

In vector tracking, traditional GPS signal tracking loops are replaced with a single filter responsible for maintaining the position, velocity, and time estimates of the receiver and for generating the local replica of the received signal used to generate error measurements. Vector tracking receivers replace the individual tracking loop filters with feedback from the navigation filter. A block diagram of a typical vector tracking architecture is shown in Figure 1. As shown in the figure, the loop filters have been removed, and the outputs of the navigation filter are used to steer the numerically controlled oscillator (NCO) to control the code and carrier frequencies of the local replicas. This design takes advantage of the fact that as the number of visible satellites increases beyond four, the navigation solution becomes over-constrained. Thus, accurate local replicas can be maintained for received satellite signals that are severely attenuated or temporarily blocked because of environmental factors. Moreover, the sharing of information from channel to channel allows for improved tracking sensitivity and improved tracking in highly dynamic situations (Lashley & Bevly, 2008).

2 | PRIOR WORK

One of the earlier vector-based carrier phase tracking approaches, called Co-OP tracking, was derived by Zhodzishky et al. in 1998. This approach uses a combination of low-bandwidth scalar tracking loops and feedback from the navigation processor. This method differs from noncoherent vector frequency tracking techniques in that the carrier phase discriminator is used to correct the navigation
solution, rather than the carrier frequency discriminator (Zhodzishsky et al., 1998). More recently, researchers at the Technical University of Munich have developed a vector phase-locked loop (VPLL) implementation that maintains phase lock by estimating position, clock, and atmospheric delay changes from epoch to epoch (Henkel et al., 2009). The algorithm requires an estimation of ionospheric delay changes for each satellite being tracked and does not directly rely on the global navigation solution. Later, the authors added vector code tracking to the VPLL in an implementation denoted “position domain joint tracking” and investigated its performance in multipath environments (Giger & Gunther, 2010; Giger & Gunther, 2011). Ohio University researchers developed a deeply integrated GPS/inertial navigation system (INS) vector phase tracking implementation. The use of an inertial measurement unit (IMU) allows the integration period to be extended well beyond the typical 20 ms up to 1 s. The authors incorporated an energy-based data bit estimator to allow for integration over bit transitions. The system was tested in-flight, in dense urban areas, and indoors, and results demonstrated carrier phase tracking for a carrier-to-noise ratio of 12–15 dB-Hz (Soloviev et al., 2007; Soloviev et al., 2008).

Many recent developments in the field of vector tracking also leverage hybrid vector/scalar strategies to improve carrier phase tracking sensitivity. Chen and Gao developed a phase-locked loop (PLL)-aided vector delay and frequency-locked loop (VDFLL) for carrier phase tracking in high dynamics (8-g lateral acceleration). The filter used an adaptive PLL in each channel and vector frequency feedback to maintain phase lock (Chen & Gao, 2019). Other work has focused on enhancing positioning accuracy by improving the reliability of carrier phase tracking. Zhifeng et al. (2022) combined a vector frequency-locked loop (VFLL) with a second-order PLL at each channel and evaluated the performance based on position accuracy. The authors demonstrated signal positioning performance at carrier-to-noise ratios near 25 dB-Hz. Improved carrier phase tracking for real-time kinematic (RTK) positioning in urban environments was developed by Humphreys et al. (2020). The tracking algorithm combined a VFLL-aided PLL with data bit wipe-off and increased integration periods to opportunistically generate carrier phase measurements with intermittent signal outages. The authors showed an 85% correct carrier integer ambiguity fix rate in a dense urban environment. Many authors have continued the development of noncoherent vector tracking approaches, with some works focused on integrity and/or multi-receiver architectures. Sun et al. (2017) used a uniformly most powerful test for interference detection and mitigation in a VDFLL navigator. Ng and Gao (2015, 2017) developed architectures combining measurements from multiple receivers into a single VDFLL navigator estimating the position of a fixed network of receivers. Two open-source vector tracking software-defined receivers have been described in the literature – both implementing a noncoherent VDFLL (Zhao & Akos, 2011; Xu & Hsu, 2019). Finally, Brewer (2014) derived a differential carrier phase tracking architecture that uses base station measurements to aid in carrier phase tracking. Carrier phase tracking results were presented based on experimental data from two static roof-top antennas at two different fixed baseline lengths.

In addition to vector tracking, direct positioning signal tracking attempts to exploit the spatial correlation of the received satellite signal by tracking in the position domain. Unlike vector tracking, direct positioning is a maximum likelihood estimation procedure in which the pseudorange and pseudorange rate residuals are not explicitly computed. Replicas are generated for candidate position, velocity, and time solutions, and the optimal solution is determined by evaluating the correlation power of each replica. This method was introduced by
Closas et al. (2007), with additional results and implementation details reported later (Closas & Gusi-Amigó, 2017). Direction positioning was originally a noncoherent tracking approach, but recent efforts have been made to extend the method to carrier phase tracking (Tang et al., 2023).

The difficulty in maintaining carrier phase lock in a true vector tracking receiver arises from the combination of the required accuracy of the navigation solution and the transient effects of atmospheric delays experienced in each channel (i.e., each satellite signal). To address these two issues, this paper uses RTK principles to maintain carrier phase lock in a truly vector-only architecture. This paper presents a novel software receiver architecture that uses fixed-integer relative position vector (RPV) estimates from an RTK algorithm and measurements from a base station receiver to predict the phase of the received carrier signal for each satellite in view. The receiver combines a vector delay-locked loop (VDLL) with the RTK-VPLL. Monte Carlo simulations are performed to compare the RTK-VPLL performance with that of traditional scalar tracking loops. In addition to the simulation study, the receiver is tested based on live sky data collected in static and dynamic tests in clear-sky and dense foliage environments. The remainder of this paper is organized as follows. First, the receiver architecture is described, with a focus on the navigation processor that maintains the RTK RPV estimates and the loop closure procedure. The Kalman filter implementation of the navigation processor is described in detail. Next, the performance of the RTK-VPLL tracking algorithm is investigated via Monte Carlo simulations. Finally, an experimental analysis of the receiver is performed by post-processing intermediate-frequency GPS samples and comparing the software receiver outputs with those of a survey-grade reference receiver.

3 | METHODOLOGY

High-precision RTK GPS receivers require continuous carrier phase lock on multiple satellite signals by the rover and base receiver. A traditional scalar tracking GPS receiver normally maintains phase lock on several signals in environments where the rover receiver has a clear line of sight to the sky and where the received signal is not disrupted by interference due to atmospheric affects or signal jammers. Applications such as precision surveying in heavy foliage and autonomous vehicle operation in urban canyons strain the capabilities of scalar tracking receivers. In this section, a software receiver vector tracking architecture is derived for improved code and carrier tracking in environments that disrupt the operations of a traditional receiver. The receiver combines a Doppler-aided VDLL with an RTK VPLL. A conceptual schematic of the system design is shown in Figure 2. The vector tracking software receiver is mounted on the rover vehicle (the tractor in the figure), and a local base station receiver is mounted in the area with a clear view of the sky. The carrier tracking algorithms of the software vector tracking receiver on the rover use measurements from the base antenna and the RPV, \( \ddot{r} \), to predict the received signal from each satellite. When measurements are available at the rover, the RPV estimates are updated.

The software receiver combines vector code and carrier tracking. In a vector receiver, the local replicas are directly driven by the navigation solution. Because of differences in the precision of the discriminator-based measurements used to update the navigation solution and possible code/cARRIER divergence due to ionospheric effects, the vector tracking receiver is designed with two navigation processors. A block diagram of the receiver architecture is shown in Figure 3.

In Figure 3, the code discriminator (\( \Delta \tau \)) and Doppler frequency (\( f_D \)) from each channel feed the VDLL navigation processor. The carrier phase discriminator (\( \delta \phi \))
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is used to update the VPLL navigation filter. The VDLL drives the code NCO, and the VPLL drives the carrier NCO. This architecture prevents the less-accurate code discriminator measurements from degrading the accuracy of the carrier phase-based navigation solution. The Doppler measurements are used to improve the velocity and clock drift state estimates in the VDLL navigation filter, similar to carrier smoothing or Doppler aiding in traditional scalar receivers. The VDLL is not a differential navigation filter; thus, the pseudorange measurements from the base station receiver are not used in the update or prediction step. This design was selected because the VDLL navigation processor is robust without the need for external data, the VDLL can maintain a navigation solution during a communication disruption because it does not rely on base station measurements, and the required communication bandwidth can be reduced by removing pseudorange measurements from the payload.

In a vector receiver, the code and carrier NCO values are set according to predictions derived directly from the navigation solution. The VDLL navigation processor

FIGURE 2 The RTK-VPLL receiver uses differential carrier phase positioning and base station measurements to improve phase tracking in degraded environments.

FIGURE 3 Block diagram of the combined VDLL and RTK-VPLL vector feedback for code and carrier replica generation, respectively.

IF: intermediate frequency
is only responsible for updating the NCO value used to generate the local code replica. A separate navigation processor is used to drive the carrier NCO. As stated earlier, this architecture was chosen to prevent degradation in the high-precision navigation solution that would result from code phase error measurement updates. A block diagram of the separation of the code and carrier generation is shown in Figure 3. In the block diagram in Figure 4, the modules used in the RTK VPLL are isolated, and the different modes of the receiver from initialization through vector operation are highlighted.

In Figure 4, the solid black lines represent the flow of information during normal (i.e., RTK-VPLL) operation, including the correlation and discrimination blocks. The black dashed lines connect the elements that are required to calculate the first receiver position and form the initial high-precision RPV (HPRPV). The RTK-VPLL initialization requires a scalar PLL in the rover receiver, measurements from the base receiver, and a differential carrier phase positioning (RTK) algorithm. The dashed line shows the outputs of the RTK algorithm that are used to initialize the RTK-VPLL, and the solid lines from the RTK-VPLL filter show the feedback used to generate the new NCO commands during normal operation. Note that there is only one RTK-VPLL Kalman filter, which receives discriminator outputs from each channel and sends NCO values to each channel.

The variables provided by the base station receiver in Figure 4 include the pseudorange ($\rho_b$), carrier phase measurement ($\phi_b$), receiver position in Earth-centered, Earth-fixed (ECEF) coordinates ($p_b$), velocity in ECEF coordinates ($v_b$), and time ($t_b$). $\rho_b$ and $\phi_b$ are mx1-dimensional arrays, where $m$ is the number of satellites that are tracked by both the base and rover receivers. The rover provides measurements of the pseudorange ($\rho_r$), carrier phase ($\phi_r$), position in ECEF coordinates ($p_r$), velocity in ECEF coordinates ($v_r$), and time ($t_r$) to the RTK processor, which computes the initialization parameters. $\rho_r$ and $\phi_r$ are mx1 arrays matching the dimensions of the base station vectors. The components of the pseudorange and carrier phase vectors from each receiver are sequentially arranged by satellite pseudorandom noise (PRN) such that the rows of each vector correspond to measurements from the same satellite. The RTK initializer provides estimates of the RPV ($r_{rb}$), relative velocity vector ($v_{rb}$), relative clock bias ($b_{rb}$), relative clock drift ($\dot{b}_{rb}$), and fixed-integer carrier ambiguities for each common channel/satellite ($N_{rb}$). Additional information on the initialization is given in Section 1 below. Note that the bold variables are vector quantities.
4 | RTK-VPLL KALMAN FILTER

The navigation filter designs in this section use code and carrier phase errors calculated from correlator outputs to update the state estimates. Therefore, it is advantageous to review the correlator output model. The mathematical model of the in-phase and quadrature correlators are given by Equation (1):

\[
\begin{align*}
I(k, \gamma) &= A(R + \gamma)D(k) \cos(2\pi f_e T + \delta\phi) + \eta_I(k) \\
Q(k, \gamma) &= A(R + \gamma)D(k) \sin(2\pi f_e T + \delta\phi) + \eta_Q(k)
\end{align*}
\]

In these equations, \(A\) is the received signal amplitude, \(\epsilon\) is the code phase error, \(f_e\) is the carrier frequency error, and \(\delta\phi\) is the carrier phase error. \(\gamma\) is the offset for the early and late replicas used to generate the code phase error measurement, and \(\eta\) is additive white noise. The coarse/acquisition (C/A) code autocorrelation function \(R(\tau + \gamma)\) is a function of the chip offset and the code phase error of the replica. The current bit of the navigation data message is given by \(D_k\). Finally, \(k\) is an index for the current integration period, and \(T\) is the integration period duration in seconds. Code phase and carrier phase error observables are used to update the VDLL and VPLL navigation processors.

A Kalman filter is used to maintain high-precision relative position estimates in the RTK-VPLL navigation processors. The state vector, shown in Equation (2), includes the three-dimensional ECEF RPV errors \((\delta x, \delta y, \delta z)\), relative velocity errors \((\delta \dot{x}, \delta \dot{y}, \delta \dot{z})\), relative clock bias error \((\delta cb)\), and relative clock drift error \((\delta c\dot{b})\):

\[
X = [\delta x \ \delta y \ \delta z \ \delta \dot{x} \ \delta \dot{y} \ \delta \dot{z} \ \delta cb \ \delta c\dot{b}]^T
\]

Note that the implementation discussed here models the receiver acceleration as driving white Gaussian noise. This model allows for easy integration with an IMU. An alternative implementation that estimates acceleration as a state can also be formulated but is outside the scope of this paper.

4.1 | Initialization

An RTK algorithm is applied to estimate the relative position, velocity, and clock states used to initialize the RTK-VPLL navigation processor. After the first fixed integer is calculated using the least-squares ambiguity decorrelation adjustment (LAMBDA) method (Teunissen, 1995), the HPRPV is calculated via the double-difference least-squares solution. The relative position states of the RTK-VPLL are initialized with the HPRPV. The relative velocity and relative clock states are initialized with the RTK Kalman filter estimates. The vector of fixed-integer single-difference carrier ambiguities, shown in Equation (3), is recorded:

\[
N_{rb} = \begin{bmatrix} N_{r,b}^1 \\
\vdots \\
N_{r,b}^m \end{bmatrix}
\]

The carrier phase ambiguities \((N_{r,b}^1, \ldots, N_{r,b}^m)\) are used as known quantities in the carrier phase prediction step of the RTK-VPLL algorithm, as described later. Recall that the superscript \(m\) is the number of satellites tracked by both the base and rover receiver. Because the relative velocity and relative clock states are estimated
in the RTK algorithm along with decimal estimates of the carrier ambiguities, the RTK-VPLL navigation solution is updated several times before the loop closure aspect of the vector PLL is initiated. The updates are performed at the update rate of the original RTK algorithm based on single-difference carrier phase measurements from the rover and base receivers. This step allows the clock bias estimate to converge to a more precise value, which may be used to predict the received carrier phase and close the phase tracking loop. More details on the RTK algorithm used for initialization have been given by Martin et al. (2010).

4.2 Time Update

The RTK-VPLL is designed with the assumption that the base receiver is stationary and has good sky visibility. Accordingly, the changes in the relative position and velocity arise from the motion of the rover receiver. Therefore, the dynamic model used in the Kalman filter time update is generally the same in the VDLL and RTK-VPLL. The discrete dynamic model used to propagate the RTK-VPLL navigation solution is shown in Equation (4):

\[
X_{k+1} = \Phi_{k,k+1} X_k + Q_k \tag{4.a}
\]

\[
\Phi_{k,k+1} = \begin{bmatrix}
\alpha_k & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} \\
0_{2 \times 2} & \alpha_k & 0_{2 \times 2} & 0_{2 \times 2} \\
0_{2 \times 2} & 0_{2 \times 2} & \alpha_k & 0_{2 \times 2} \\
0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} & \alpha_k \\
\end{bmatrix} \tag{4.b}
\]

\[
\alpha_k = \begin{bmatrix}
1 \\
\Delta t \\
0 \\
1 \\
\end{bmatrix} \tag{4.c}
\]

\[
Q_k = \begin{bmatrix}
Q_x & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} \\
0_{2 \times 2} & Q_y & 0_{2 \times 2} & 0_{2 \times 2} \\
0_{2 \times 2} & 0_{2 \times 2} & Q_z & 0_{2 \times 2} \\
0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} & 2Q_{cb} \\
\end{bmatrix} \tag{4.d}
\]

\[
Q_x = Q_y = Q_z = \begin{bmatrix}
\frac{\sigma^2 \Delta t^3}{3} & \frac{\sigma^2 \Delta t^2}{2} \\
\frac{\sigma^2 \Delta t^2}{2} & \sigma^2 \Delta t \\
\end{bmatrix} \tag{4.e}
\]

\[
Q_{cb} = \begin{bmatrix}
\frac{\sigma_b^2 T^3}{3} + \frac{\sigma_r^2 T^3}{2} & \frac{\sigma_r^2 T^2}{2} \\
\frac{\sigma_r^2 T^2}{2} & \sigma_r^2 T \\
\end{bmatrix} \tag{4.f}
\]

Equation (4) is derived from the kinematic relationship of the states and assumes that the velocity states are driven by zero-mean Gaussian white noise. The state transition matrix \((\phi_{k,k+1})\) and the process noise matrix \((Q_k)\) are defined in Equations (4.b)–(4.f). Equation (4.e) shows that the variance on the platform acceleration \((\sigma^2)\) is assumed to be the same in all directions. In practice, it may be
possible to use platform constraints when selecting these values. The acceleration variance ($\sigma^2$) is a tunable parameter, based on the expected platform motion. In the simulation results presented below, several specific acceleration variance values are tested. $Q_{cb}$ represents the stochastic components of the clock bias and drift models, as described by Brown and Huang (2012). Note that the process noise matrix (Equation (4.d)) reflects the fact that the relative clock bias and drift include the effects of both the rover and base clocks. Equation (4.f) assumes that the clock model parameters (e.g., $\sigma_b^2$, $\sigma_r^2$) are the same for the rover and base clocks and that the stochastic errors are uncorrelated. In the simulation and experimental results shown later, a temperature-compensated oscillator is assumed, with $\sigma_b^2 = 1.258 \times 10^{-6}$ and $\sigma_r^2 = 4.258 \times 10^{-5}$. These values were determined experimentally for a hardware oscillator used for experimental testing.

### 4.3 Measurement Update

The navigation solution of the RTK-VPLL is initialized with position information that is accurate enough to predict the received carrier phase. To maintain this level of accuracy, the Kalman filter residuals must be as accurate as the single-difference carrier phase measurements used to estimate the HPRPV. The carrier phase discriminator provides a range error measurement with an accuracy of a few millimeters. The RTK-VPLL Kalman filter residual is shown in Equation (5):

$$\delta \phi = \lambda_{L1} \tan \left( \frac{Q_P}{I_P} \right)$$

Here, $I_P$ and $Q_P$ are the in-phase and quadrature prompt correlator outputs, which are defined based on Equation (1) as $I(k,0)$ and $Q(k,0)$, respectively. The Costas carrier discriminator is used to calculate the residual carrier phase error, and the wavelength of the GPS L1 signal ($\lambda_{L1}$) is used to convert to units of meters. Note that this paper is specifically focused on the GPS L1 C/A code only, but it is the author’s opinion that the algorithm will generalize well to other global navigation satellite system signals.

The primary error source in the carrier tracking loop is thermal noise. Accordingly, the measurement uncertainty of the carrier discriminator is calculated as a function of the carrier-to-noise ratio of the signal from each channel. Carrier phase error measurement noise is modeled based on the suggestion of McGraw and Braash (1998), as given in Equation (6):

$$\sigma_{\phi}^2 = \frac{\lambda_{L1}}{2\pi \Delta \phi} \left( \frac{C}{N_0} \right)$$

In Equation (6), $\frac{C}{N_0}$ is the carrier-to-noise ratio, which is computed from the amplitude of the prompt correlator and the sample noise, as described by Lashley (2009). A coherent integration period ($T$) of 20 ms is used for improved tracking in degraded signal environments.

The RTK-VPLL measurement updates are performed at the end of the integration and dump period for each channel. The integration periods align with whole bits of the navigation message. Therefore, the measurement updates for the channels
occur asynchronously. The measurement vector only includes one of the $m$ total carrier phase discriminators at a time, as shown in Equation (7):

$$z = \lambda_{11}[\delta\phi]$$

(7)

Note that the filter must account for changes in the state vector during an integration period caused by other channels. The modification needed to account for the asynchronous update has been described in detail by Lashley & Bevly (2011).

The measurement matrix ($H$) maps the state errors into the measurement domain using the line-of-sight unit vectors ($a_x, a_y, a_z$) from satellite to receiver. This matrix is defined in Equation (8):

$$H = [a_x \ 0 \ a_y \ 0 \ a_z \ 0 \ -1 \ 0]$$

(8)

Recall that the state vector includes estimates of the relative velocity vector (i.e., rover velocity because the base station is assumed to be static) and the relative clock drift. The zeros in the $H$ matrix arise from the assumption that the velocity errors and relative clock drift do not map the carrier phase residual. The column of negative one relates the relative clock error to the carrier phase residual.

The Kalman filter measurement update is performed using the traditional expressions given in Equation (9):

$$K_k = P_k H^T (H P_k H^T + R_k)^{-1}$$

(9.a)

$$P_k = (I - K_k H) P_k$$

(9.b)

$$X_k = X_k + K_k z_k$$

(9.c)

Note that the $R_k$ matrix is scalar within each channel update and $R_k = \sigma^2$, as described in Equation (6). The Kalman filter time and measurement equations were originally derived by Kalman (1960), and implementation details for GPS positioning were described by Brown and Huang (2012).

### 4.4 Carrier Loop Closure

To close the carrier tracking loop, the RTK-VPLL state estimates and the carrier phase and Doppler measurements from the base station receiver are used to predict the carrier phase of the received signal at the end of the current integration period. The current integration period refers to the integration period that begins directly after the VPLL measurement update. The prediction step begins with propagation of the base station carrier phase from the time of the measurement to the end of the current integration period. The updated base station carrier phase is calculated from Equation (10):

$$\hat{\phi}_{b,t+1} = \hat{\phi}_b + \omega_{b} (t_{k+1} - t)$$

(10)

The $k$ notation is not used with the base station carrier phase measurement because the base station measurements are synchronized to GPS time and are not synchronous with the end of an integration period in the rover tracking channels. In Equation (10), $\hat{\phi}_b$ is the most recent base station carrier phase measurement, $\omega_{b}$ is the most recent Doppler measurement from the base station, $t_{k+1}$ is the
receiver time at the end of the current integration period, and $t$ is the time of the latest base station measurements.

Next, the RTK-VPLL navigation states are projected forward to time $t_{k+1}$ based on the state transition matrix defined in Equation (4.b). The error state mean and covariance are not updated at this time. The predicted states are used along with the base-receiver-predicted carrier phase and the carrier ambiguity to calculate the predicted carrier phase at the rover at time $t_{k+1}$ according to Equation (11):

$$
\hat{\phi}_{k+1} = \hat{\phi}_{b_{k+1}} + \frac{1}{2L_1} \left( \hat{a}_{k+1} \hat{r}_{r,b_{k+1}} + \hat{c}_{r,b_{k+1}} \right) + N_{r,b}
$$

(11)

The predicted state estimates are used to calculate the line-of-sight phase difference between the rover and base represented in Equation (11) by the middle term on the right-hand side (i.e., the term in parentheses). Note that $\hat{a}_{k+1}$ is the three-dimensional line-of-sight unit vector, $\hat{r}_{r,b_{k+1}}$ is the predicted three-dimensional RPV, and $\hat{c}_{r,b_{k+1}}$ is the predicted relative clock bias. As stated in Section 1, an RTK algorithm is used to fix the carrier phase ambiguities to integers prior to the start of the RTK-VPLL. Therefore, the carrier ambiguity ($N_{r,b}$) is assumed to be known in Equation (11) and is assumed to be constant after initialization. This assumption can lead to positioning inaccuracies in the case that a cycle slip occurs, as will be shown later in the simulation and experimental results.

The total phase change during the integration period includes both the Doppler effect and the intermediate frequency of the GPS front-end. To calculate the desired carrier frequency setting for the carrier NCO, the total phase change over the integration period is divided by the change in receiver time, as defined in Equation (12):

$$
\omega_\theta = \frac{\omega_{IF} \left( t_{k+1} - t_k \right) - \left( \hat{\phi}_{k+1} - \hat{\phi}_b \right)}{t_{k+1} - t_k}
$$

(12)

In this equation, $\omega_{IF}$ is the intermediate frequency of the GPS front-end, and $\hat{\phi}_b$ is the current carrier phase measurement of the rover receiver. Note that the change in carrier phase is subtracted because the Doppler frequency decreases as the range between the transmitter and receiver increases.

5 | SIMULATION RESULTS

The vector tracking software receiver combines the Doppler-aided VDLL with the RTK VPLL to provide robust high-precision positioning. Several analyses of VDLL code tracking receivers are available in the literature (Lashley & Bevly, 2008; Pany & Eissfeller, 2006; Clark, 2012), whereas the phase tracking of a software receiver is the focus of this paper. Therefore, the present analysis of the software vector tracking receiver focuses on the performance of the carrier phase tracking algorithms. Simulation analysis results on the thermal noise tracking performance of the receiver are presented in this section.

A correlation-level simulation of the RTK-VPLL was developed to evaluate performance. The simulator generated in-phase and quadrature early, prompt, and late correlator outputs based on Equation (1). The VDLL runs parallel to the RTK-VPLL, as it would in a real implementation. The simulation tool was used to analyze the performance of the RTK-VPLL algorithm as a function of the carrier-to-noise ratio of the received signal. The simulator is designed to generate correlator outputs that are representative of actual correlator outputs, accounting
for the carrier-to-noise ratio of the simulated signal and the errors in the local replicas generated by the code and carrier NCOs. Three different acceleration variances were simulated to analyze the response of the receiver to variable platform dynamics. The acceleration was modeled as a zero-mean Gaussian random variable with a standard deviation of either 0, 1, or 3 m/s². The three scenarios represent a static platform, a slow-moving platform similar to a pedestrian or small robot, and a moderately dynamic platform similar to a ground vehicle. As stated in the section on the RTK-VPLL Kalman filter, the current filter implementation models acceleration as a Gaussian random variable in part because this model allows for easy IMU integration. The acceleration variance used in the simulation also provides some insight into the impact of INS acceleration uncertainty on the RTK-VPLL in a deeply coupled GPS/INS integration. The carrier-to-noise ratio of the received signal was held constant during each simulation, and all channels were simulated with the same ratio. Simulations were performed with carrier-to-noise ratios ranging from 20 dB-Hz to 30 dB-Hz. During the simulations, it was assumed that the base station receiver had a clear line of sight to the satellites and that the received signals were relatively strong. The carrier-to-noise ratios of the signals received by the base station receiver were set to 45 dB-Hz. The integration periods for the RTK-VPLL and VDLL were 20 ms. Twenty Monte Carlo runs were performed for each combination of acceleration variance and carrier-to-noise ratio. The results of the simulations are summarized in the following figures and table.

First, the static receiver scenario is analyzed. The two plots in Figure 5 show the carrier phase tracking error for each satellite signal for 5 of the 20 Monte Carlo simulations. Note that ten satellite signals were simulated in each Monte Carlo run. The two plots each show 50 channels (10 satellites × 5 simulations) of carrier phase error plotted versus simulation time. The plots are limited to 50 channels for readability. The 50 channels were selected to contain approximately the same percentage of locked and unlocked channels as the total data set. The carrier phase errors for each channel are all shown in black to avoid confusion. Each plot shows the carrier phase error for one of the selected carrier-to-noise ratios: 22 dB-Hz and 21 dB-Hz. The carrier-to-noise ratio is shown in the title of each plot for clarity.

Based on the plot corresponding to a carrier-to-noise ratio of 22 dB-Hz, the RTK-VPLL was able to maintain phase lock on all signals when a carrier-to-noise ratio of 22 dB-Hz was simulated. Note that the carrier phase errors for all

FIGURE 5 RTK-VPLL carrier phase error as a function of simulation time for zero acceleration, showing the comparison of tracking performance for the carrier-to-noise ratios of 22 dB-Hz (left) and 21 dB-Hz (right)
channels are less than one cycle. Alternatively, the plot corresponding to a simulated carrier-to-noise ratio of 21 dB-Hz shows that the carrier phase error for several channels diverges in excess of 6 cycles. In fact, the carrier phase error for these channels continues to increase well beyond 6 cycles. The plot is constrained to a range of −6 to 6 cycles for direct comparison with the 22-dB-Hz plot. In simulation testing, the carrier phase error and position error continue to grow unbounded once the RTK-VPLL loses its lock.

For comparison, a scalar tracking PLL was implemented and tested using the same simulation settings. The PLL used a third-order loop filter with a noise bandwidth of 12 Hz. The filter was designed based on the work of Kaplan and Hegarty (2017), and the noise bandwidth was selected after testing values between 8 and 18 Hz. A noise bandwidth of 12 Hz consistently provided the best performance. Figure 6 shows the range of carrier-to-noise ratios at which the scalar PLL begins to lose lock. With the scalar PLL, the receiver was able to track at a carrier-to-noise ratio of 29 dB-Hz, as shown in the figure corresponding to that signal power level (see plot title for carrier-to-noise ratio). Cycle slips and carrier divergence begin at a carrier-to-noise ratio of 28 dB-Hz, as shown in the corresponding plot. Compared with the RTK-VPLL results shown above, the scalar tracking PLL requires a carrier-to-noise ratio approximately 7 dB-Hz higher to maintain phase lock.

The tracking performance of the RTK-VPLL architecture is dependent on the accuracy of the navigation solution. To maintain phase lock, the RTK-VPLL position solution must be accurate to within approximately 5 cm, which represents approximately one quarter of the GPS L1 wavelength. As the positioning error approaches half the wavelength, we see cycle slipping in the tracking loops. This cycle slipping can lead to filter divergence in some scenarios. In Figure 7, the ECEF position error of the RTK-VPLL navigation processor is shown for the simulations displayed in Figure 5. The navigation solution is clearly accurate enough to maintain phase lock when the carrier-to-noise ratio at the rover is 22 dB-Hz. When the carrier-to-noise ratio drops to 21 dB-Hz, the navigation solution rapidly degrades in three of the five simulations shown. Again, the plot is bounded within −0.1 to 0.1 m of position error to allow for direct comparison to the plot for 22 dB-Hz. As stated previously for the tracking results, the navigation solutions that diverge continue to grow unbounded after the initial failure. The three simulations that resulted in failure show the propensity of the receiver to fail catastrophically when it does.
fail. This result is due to the fact that an error in one channel will affect all other channels. Unfortunately, at low carrier-to-noise ratios, it is difficult to distinguish faults in the carrier phase discriminators. Despite this weakness, the RTK-VPLL receiver shows considerable improvement in thermal noise performance over traditional tracking approaches. Note that there were two simulations in which the RTK-VPLL tracking algorithm was able to maintain an accurate navigation solution at 21 dB-Hz. However, the majority of the simulations at 21 dB-Hz resulted in catastrophic failure.

Figure 8 shows the impact of adding receiver dynamics to the simulations. A modest random acceleration with a variance of $1 \text{m/s}^2$ was added for this simulation. As shown in Figure 8, the RTK-VPLL performance is degraded slightly by the receiver motion. This result occurs because the process noise matrix values must be increased slightly to account for the unknown dynamics. As a result, the steady-state Kalman gain is increased, yielding a high effective noise equivalent
bandwidth of the vector loop filter. Under these conditions, phase lock was lost on two of the five simulations at a carrier-to-noise ratio of 22 dB-Hz, as shown in the plot titled $C/N_0 = 22$ dB-Hz. In one of the five simulations plotted on the left (titled $C/N_0 = 23$ dB-Hz), the receiver experienced a cycle slip for a carrier-to-noise ratio of 23 dB-Hz. This cycle slip did not cause catastrophic failure of the RTK-VPLL tracking loop.

The impact of the cycle slip on the navigation solution is shown in Figure 9. In the plot on the left (titled $C/N_0 = 23$ dB-Hz), one of the simulation results shows an increase in the ECEF x, y, and z position errors starting at approximately 1.5 s into the simulation. The solution does not diverge, but the error settles to approximately 0.5 cm in the x direction and approximately 2 cm in the y and z directions. In this case, the RTK-VPLL carrier phase prediction is accurate in terms of the fractional phase, but the carrier ambiguity has increased by one cycle on all channels. This result indicates that the RTK-VPLL receiver is capable of operating with small errors in the fixed carrier phase ambiguities at the expense of increased navigation solution errors. Similar to the carrier phase error plots in Figure 8, it is clear in the plot on the right (titled $C/N_0 = 22$ dB-Hz) that the navigation solution has diverged from the true RPV, resulting in a loss of phase lock in two of the five simulations at a carrier-to-noise ratio of 22 dB-Hz.

As the platform accelerations increase, the minimum carrier-to-noise ratio at which the RTK-VPLL receiver can track increases. For brevity, simulation results for moderate acceleration are not presented individually. The combined results of the three simulation scenarios are summarized below.

To analyze the loss of the phase lock condition of the RTK-VPLL algorithm, the accuracy of the Kalman filter RPV estimates are examined in the measurement domain. For the carrier phase discriminator from each channel to provide accurate phase error information, the carrier NCO must provide a carrier replica that is accurate within the range of the discriminator (one quarter cycle for the Costas phase discriminator). The RPV errors are estimated in ECEF coordinates and are mapped onto the line-of-sight vector from the receiver to the satellite based on the unit vectors calculated in the Kalman filter. It is assumed that the RTK-VPLL algorithm will maintain phase lock as long as four or more local carrier replicas
are generating accurate carrier phase discriminators. Consequently, the RPV error mapped into the measurement domain must be less than one quarter cycle for at least four satellite signals that are being tracked. Loss of lock occurs when fewer than four channels have line-of-sight phase errors that are less than one quarter cycle. To evaluate the RTK-VPLL loss-of-lock performance, Figure 10 shows the number of channels with errors greater than one quarter cycle for the three dynamic scenarios tested in simulation. Each plot shows 5 of the 20 simulations for each scenario. The results for the critical carrier-to-noise ratio (i.e., the maximum carrier-to-noise ratio at which a loss of lock is prevalent) are presented for each dynamic scenario. Note that ten satellites are tracked in each simulation. Thus, the loss of seven or more channels indicates a loss of phase lock for the vector receiver. In the figure, each color represents a different Monte Carlo simulation. The primary observation from these three plots is that a loss of lock on individual channels compounds rapidly and affects the other tracking channels, especially when the signal strength on each channel is equivalent. Recall that the carrier replicas are generated directly from the navigation solution. Therefore, a bad measurement on one channel can cause an increase in navigation solution error and degrade the replicas generated on all other channels. Ideally, the Kalman filter would de-weight bad measurements because the measurement covariance would be high (because of a low carrier-to-noise ratio). Unfortunately, the state covariance will be high when all channels have low carrier-to-noise ratios, causing the filter to be overly trusting of the carrier phase residuals. In many practical applications, the

FIGURE 10  RTK-VPLL lock detection for zero, low, and moderate acceleration at carrier-to-noise ratios near the limit of tracking capability shows rapid channel failure after the first channels lose lock.
carrier-to-noise ratio will vary from channel to channel, thus allowing the channels that are tracking strong signals to aid the channels that are tracking weak signals. For the case in which strong signals are present, the filter will be able to better filter the weak signals because the state covariance will not be as high.

For each simulated carrier-to-noise ratio, the standard deviation of the phase error was calculated for the three different acceleration models. The results are displayed in Figure 11. This figure gives an indication for the carrier phase tracking capability of the RTK-VPLL for each scenario. The black line in the figure represents an approximate tracking threshold, which assumes that phase lock can be maintained with a phase error standard deviation below 15° (Kaplan & Hegarty, 2017). Clearly, increasing the standard deviation of the simulated acceleration degraded the performance of the RTK-VPLL receiver. At low dynamics, the RTK-VPLL receiver was able to maintain phase lock at significantly lower carrier-to-noise ratios than the traditional scalar PLL receiver (approximately 28 dB-Hz, as shown earlier). The improved tracking sensitivity was more modest for higher accelerations. Figure 11 shows that the RTK-VPLL receiver can reliably maintain phase lock at carrier-to-noise ratios of approximately 22–23 dB-Hz with low platform dynamics.

The statistical results of the RTK-VPLL thermal noise performance analysis are compiled in Table 1. Included in the table are the carrier-to-noise ratios that

![Figure 11](image-url)

**Figure 11** Standard deviation of carrier phase error of the RTK-VPLL receiver as a function of carrier-to-noise ratio and platform acceleration, showing phase tracking limits relative to a 15° tracking threshold

<table>
<thead>
<tr>
<th>C/N₀</th>
<th>σᵢ = 0 m/s²</th>
<th>σᵢ = 1 m/s²</th>
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<td>26</td>
<td>9.3 σᵣ % Lost</td>
<td>10.0 σᵣ % Lost</td>
<td>15.6 σᵣ % Lost</td>
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<td>10.3 σᵣ % Lost</td>
<td>11.2 σᵣ % Lost</td>
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<td>12.3 σᵣ % Lost</td>
<td>18.6 σᵣ % Lost</td>
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<tr>
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<td>14.5 σᵣ % Lost</td>
<td>23.8 σᵣ % Lost</td>
</tr>
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<td>21</td>
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<td>NA 100</td>
</tr>
<tr>
<td>20</td>
<td>NA 100</td>
<td>NA 100</td>
<td>NA 100</td>
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</table>

<table>
<thead>
<tr>
<th>C/N₀</th>
<th>σᵢ = 0 m/s²</th>
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</thead>
<tbody>
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<tr>
<td>29</td>
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<td>27</td>
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<td>26</td>
<td>260.0 σᵣ % Lost</td>
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<td>25</td>
<td>314.3 σᵣ % Lost</td>
</tr>
<tr>
<td>24</td>
<td>343.3 σᵣ % Lost</td>
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</table>

<table>
<thead>
<tr>
<th>C/N₀</th>
<th>σᵢ = 0 m/s²</th>
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<tbody>
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</tr>
<tr>
<td>25</td>
<td>100 NA 100</td>
</tr>
<tr>
<td>24</td>
<td>100 NA 100</td>
</tr>
</tbody>
</table>

**Table 1**

RTK-VPLL Thermal Noise Performance Simulation Results with Scalar Comparison
were simulated, the phase standard deviations in degrees, and the percentage of simulations for which the vector phase lock was lost. Results are shown for the simulations with a static platform, low platform dynamics, and moderate platform dynamics. Overall, the RTK-VPLL phase tracking performance degrades slightly as the receiver dynamics increase, as shown in the previous figures. For lower dynamic scenarios, the receiver maintains phase lock at carrier-to-noise ratios as low as 22 dB-Hz. Note that the RTK-VPLL receiver failed at 23 and 24 dB-Hz in some static scenarios. This failure occurred in a minority of the simulations and must be attributed to the stochastic nature of the simulation. These failures typically occurred during the initial transient phase of the filter operation. Based on the consistent degradation of the RTK-VPLL in the dynamic scenarios, it is expected that additional Monte Carlo simulations (i.e., significantly more than 20 runs) would likely show that failures occur with a lower frequency than shown here. The table also includes the scalar PLL tracking results for static simulation scenarios. As shown earlier, the scalar PLL begins losing lock at approximately 28 dB-Hz, and the carrier phase error standard deviation increases rapidly for carrier-to-noise ratios lower than 28 dB-Hz. In the dynamic simulations, the scalar PLL performance degraded at approximately the same rate as the RTK-VPLL.

6 | EXPERIMENTAL RESULTS

Intermediate-frequency GPS data were collected in a variety of conditions to analyze the performance of the RTK-VPLL vector tracking receiver in real-world settings. Three clear-sky tests, two static and one dynamic, were conducted to analyze the accuracy of the differential carrier phase-based navigation solution in comparison to an RTK reference solution. Data were also collected as the test vehicle traveled near the Auburn University campus in areas where tree cover and buildings affected the sky visibility. Degraded GPS environment tests were conducted in areas that are designated as moderate- or heavy-foliage environments. The moderate-foliage environments are primarily characterized by tree-lined multi-lane roads for which the sky view was regularly disrupted by trees, particularly to one side of the vehicle. In the heavy-foliage environments, large portions of the sky were blocked by dense tree cover on narrow neighborhood streets.

The first experimental analysis of the RTK-VPLL architecture was performed under ideal conditions. The signal from one antenna mounted on the roof of the Woltosz Engineering Building was recorded by two GPS front-ends. The sample clocks of the front-ends were synchronized to a common reference signal, and data were collected for approximately 10 min. The intermediate-frequency samples were processed using a vector tracking software receiver that incorporated the VDLL and RTK-VPLL code and carrier tracking loops. In this scenario, the true RPV and relative clock drift are known to be zero. Therefore, it is straightforward to analyze the accuracy of the relative position and relative clock estimates. The error statistics are presented later in Table 2, along with the other clear-sky results.

A second static test was performed to analyze the RTK-VPLL accuracy over a longer separation distance. The base station and rover receivers were located approximately 600 m apart, with a vertical displacement of approximately 30 m. The reference RPV was calculated by averaging 10 min of high-precision RTK position solutions from a survey-grade GPS receiver. The RTK-VPLL algorithm was initialized with the first integer fixed from the scalar RTK algorithm, as described above. The RPV estimation error in the horizontal and vertical directions is presented graphically in Figure 12. The errors in the east and north directions are shown in the scatterplot on the left in units of centimeters. Clearly, the accuracy
### Table 2
**RTK-VPLL Experimental Error Statistics**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Separation Distance</th>
<th>Mean Error (m)</th>
<th>Standard Deviation (m)</th>
<th>RMSE (m)</th>
</tr>
</thead>
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<td></td>
<td>0 m</td>
<td>600 m</td>
<td>Variable</td>
<td></td>
</tr>
<tr>
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<td></td>
<td>5.63x10^{-4}</td>
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<td>1.76x10^{-3}</td>
</tr>
<tr>
<td>North</td>
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<td>-1.34x10^{-3}</td>
<td>7.37x10^{-3}</td>
<td>7.66x10^{-3}</td>
</tr>
<tr>
<td>Up</td>
<td>-1.42x10^{-3}</td>
<td>-1.61x10^{-2}</td>
<td>9.55x10^{-3}</td>
<td>1.07x10^{-2}</td>
</tr>
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<td></td>
<td></td>
<td>600 m</td>
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<td></td>
<td></td>
<td>Variable</td>
<td>6.85x10^{-3}</td>
<td>4.76x10^{-2}</td>
</tr>
<tr>
<td></td>
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<td>5.38x10^{-5}</td>
<td>1.89x10^{-2}</td>
<td></td>
</tr>
<tr>
<td></td>
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<td></td>
<td></td>
<td>4.32x10^{-3}</td>
<td>1.89x10^{-2}</td>
<td></td>
</tr>
</tbody>
</table>

The error is again at the centimeter level, with a mean of approximately zero. The relative vertical position estimates are also accurate to within a few centimeters but show a slight bias relative to the reference solution. Note that the NovAtel reference receiver is a dual-frequency (L1 and L2) receiver, which may contribute to the offset in the vertical direction.

An additional experiment was performed to assess the accuracy of the RTK-VPLL navigation solution accuracy during motion in a clear-sky environment. Data were collected using an intermediate-frequency data recorder while the test vehicle traveled at slow speeds around an open field. The vehicle was driven at speeds ranging from 3 to 5 m/s around the field for approximately 6 min. The base station receiver was mounted on the roof of a static vehicle location at the southwest corner of the test site. The separation distance between the base station and the rover receiver varied during the test from 5 to 210 m.

A qualitative comparison of the RTK-VPLL navigation solution and the reference RTK solution is provided in Figure 13. This figure shows the NovAtel RTK reference solution along with the RTK-VPLL position estimates for the vehicle during the first turn of the dynamic test. The figure also shows the intersection of three different laps made by the vehicle around the gravel path. In all three cases, the
reference position estimates and the RTK-VPLL position estimates are virtually identical on this scale.

Statistics for the position estimation errors of the RTK-VPLL receiver, as compared with the RTK reference solution, are compiled in Table 2. The mean and standard deviations of the errors are shown for the signal antenna, 600-m baseline, and dynamic tests. As shown in the table, the mean horizontal errors in the east and north directions are sub-centimeter in both directions for all three tests. The mean vertical error was just over 1 cm in magnitude for the longer baseline and dynamic tests. The standard deviations of the errors are also centimeter-level or better. The root mean squared error (RMSE) of the RTK-VPLL solution relative to the reference solution is also provided in the table. The RMSE is sub-centimeter in all scenarios, with the exception of the north and up directions in the dynamic test. The accuracy of the software vector tracking receiver operating in RTK-VPLL mode is consistent with survey-grade GPS receivers using RTK corrections from a local base station.

To further evaluate the capabilities of the software vector tracking receiver, data were collected as the test vehicle traveled around the Auburn University campus. The test route traversed areas in which the line of sight to the sky was disrupted by tree cover and buildings. The reference antenna was located on the roof of the Woltosz Engineering Building in the north central portion of campus. The separation distance between the rover and base station antennas ranged from approximately 100 m to 1 km. The maximum vehicle speed was approximately 14 m/s. The roads of the test route were mostly tree-lined multi-lane roads for which the sky view of the antenna was obstructed by trees up to 75° above the horizon.

Figure 14 shows the complete test route. The orange dots show the position solution as calculated by the software receiver using the RTK-VPLL algorithm. The solution is clearly continuous throughout the route, following the roads driven during the test. The blue dots (labeled NovAtel HP) show the high-precision RTK reference solution (i.e., fixed-integer RTK solution). There are several areas for which the reference receiver was unable to maintain phase lock on enough satellite signals to produce a high-precision position solution. In some areas, the reference solution appeared to be degraded, possibly because of multipath signals corrupting the carrier phase measurements. The route begins in the northwest corner and includes a loop that comprises the western part of the route and the eastern part...
of the route, which ends in the southeast corner. The western part of the route travels through the Auburn University campus and contains moderate foliage and the Jordan-Hare Stadium, which obstructs nearly half the sky view for the receiver. The eastern part of the route is more suburban, with heavy foliage often covering large portions of the sky. These two sections will be referred to as moderate- and heavy-foliage regions in the following discussion of results.

A closer view of the RTK-VPLL navigation solution is provided in Figure 15, which shows the lane-level accuracy of the RTK-VPLL navigation solution. The tree cover at the intersection prevents the reference from reporting a high-precision solution. Note that the antenna is mounted on the driver side of the vehicle just outside the frame of the vehicle and that the vehicle is driving north in the image. The antenna-mounting location is consistent with the location of the navigation solution relative to the satellite imagery shown in Figure 15. The NovAtel pseudo-range (labeled standalone) position solution and the VDLL position solutions are also included in the plot in an attempt to highlight the driven route.

The robustness and accuracy of the RTK-VPLL navigation solution is shown in Figure 16. The estimated position of the vehicle remains within the lane, as shown by the satellite imagery as the vehicle moves along a densely wooded street. As noted previously, the antenna was mounted on the driver’s side of the vehicle. In the image, the position estimates appear to trace the left edge of the lane as the vehicle moves to the south. The reference receiver was unable to calculate a pseudorange-based navigation solution in this environment and had not reported a high-precision carrier phase estimate for several hundred meters by this point.

For the weakest signal environments in which tree cover blocks most of the sky, the RTK VPLL receiver was not able to maintain the phase of the received
FIGURE 15 The RTK-VPLL navigation filter provided lane-level accuracy through intermittent heavy tree cover whereas the reference system could not produce an RTK solution. Imagery by Google, DigitalGlobe.

FIGURE 16 The RTK-VPLL navigation filter provided accurate positioning through densely wooded neighborhood streets whereas the reference receiver could not provide an RTK solution. Imagery by Google, DigitalGlobe.
signals without carrier cycle slips. The errors in the RTK VPLL navigation solution resulting from the cycle slips are apparent in Figure 17. In this figure, the vehicle travels southeast before turning right onto a densely wooded narrow street (near latitude 32.5895 and longitude -85.48). The vehicle is driven through a parking lot, as shown on the far west portion of the route, where the reference receiver briefly reports a solution. Then, the vehicle is turned back to the east and returns to the original street traveling southeast. The vehicle drives on another loop through a second parking lot before again returning to the original street (the second loop begins and ends near latitude 32.5885 and longitude -85.479). In the heavy-foliage area, the carrier phase discriminators used by the RTK-VPLL Kalman filter to update the navigation solution do not provide the needed accuracy to maintain centimeter-level precision. There is some improvement in the navigation solution as the vehicle moves away from the densest foliage, as shown for the southeastern section in the figure (near latitude 32.587 and longitude -85.478). The position estimates return to the road, as shown on the satellite imagery. Note that the high-precision NovAtel solution (i.e., fixed-integer RTK solution) was unavailable throughout this part of the route because the receiver was not able to maintain carrier phase lock on at least four satellites.

To further analyze the experimental performance of the RTK-VPLL receiver, the correlator outputs from each channel were used to calculate a phase lock indicator, as described by Spilker (1996). The phase lock indicator is a ratio of the relative power in the in-phase and quadrature prompt branch to the total power of the prompt correlators. If the phase lock indicator exceeds the lock threshold, the channel is considered phase-locked. For additional details on the phase lock indicator, see the work by Spilker (1996). The phase lock results for the moderate-foliage test are shown in Figure 18. In the image on the left, the number of locked channels
is plotted as a function of time. The number of locked channels in the reference receiver is also shown for comparison. This figure clearly shows that the RTK-VPLL often has a lock on a larger number of channels, particularly at times when the conditions are difficult. Note that the maximum number of locked channels does not increase above the original number of locked channels for the RTK-VPLL receiver because the receiver does not continue to run acquisition after startup.

The length of time for each outage of the RTK VPLL receiver is also shown in Figure 18. An outage is defined as a measurement interval (20 ms) during which fewer than four channels pass the phase lock test. In this figure, the longest outage lasts approximately 100 ms, and most outages are less than 50 ms. Cycle slips are more likely to occur during longer outages.

The phase lock performance and outage lengths for the RTK-VPLL receiver operating in the heavy-foliage environment are shown in Figure 19. A comparison of Figures 18 and 19 shows that the heavy-foliage environment had a more dramatic effect on tracking performance. There are times during the test when only one channel in the RTK-VPLL receiver passes the phase lock test. The reference receiver often only maintains phase lock on zero or one channel. The outage length for the RTK VPLL receiver increases significantly in the heavy-foliage environment, which is reflected by the qualitative results shown previously, where a bias in the navigation solution suggests numerous cycle slips. The longest outage is nearly 1 s,
which corresponds to almost 50 iterations when the navigation solutions are not fully constrained (i.e., four or more phase-locked channels).

In Table 3, the phase lock performances of the reference receiver and RTK-VPLL receiver are quantitatively characterized by the percentage of time during each test that each channel did not pass the phase lock test. The results are shown for six satellites that were tracked by each receiver during one or both tests. As expected, each receiver performed better during the moderate-foliage test. On average, the RTK-VPLL receiver lost lock 1.9% of the time in the moderate-foliage environment, compared with 9.5% of the time in the heavy-foliage environment. The RTK-VPLL receiver maintained lock more often (i.e., lower percentage of time not locked) than the reference receiver on every channel during both tests. The improvement ranged from 2% to 58%.

The phase lock indicator gives some insight into the quality of the discriminator outputs provided by each channel, but it does not necessarily indicate that the vector tracking receiver has lost lock on a channel. The carrier replica is generated by the navigation processor; therefore, the receiver can accurately generate carrier replicas if the navigation solution is accurate enough to predict the phase to within the effective range of the discriminator. For instance, let us consider the case in which the receiver is tracking six satellite in vector mode, and at some point, two of those signals are blocked by obstructions. The states of the navigation filter are still observable with the measurements from the remaining four satellite signals. Therefore, the navigator maintains a relative position solution with centimeter-level accuracy and accurately predicts the blocked signals, even though that channel is not providing any information. Later, the obstructions are removed, and the two signals come back into view. As long as navigation accuracy has been maintained, the replicas on those two channels are instantaneously re-locked. In fact, they were never truly unlocked. A vector receiver is often thought of as permanently locked as long as the required navigation solution accuracy is maintained.

Let us consider the azimuth and elevation plot for the six satellites tracked during the foliage tests, as shown in Figure 20. The signals from satellite 3 and satellite 28 are likely to be obstructed because of their height above the horizon. A covariance analysis was performed to show the observability of the navigation solution when only measurements from satellites 2, 6, 17, and 19 were used in the measurement update. The line-of-sight uncertainty was calculated based on the satellite-to-receiver unit vectors and the state covariance matrix.

The results are shown in Figure 21. In this figure, the line-of-sight phase uncertainty corresponding to one standard deviation is shown as a function of time. At the 20-s mark, the number of measurements used in the correction step of the Kalman filter was reduced from six to four measurements to simulate an outage of

<table>
<thead>
<tr>
<th>PRN</th>
<th>Moderate Foliage</th>
<th>Heavy Foliage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NovAtel</td>
<td>RTK-VPLL</td>
</tr>
<tr>
<td>2</td>
<td>45.7</td>
<td>2.5</td>
</tr>
<tr>
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<tr>
<td>28</td>
<td>42.5</td>
<td>3.6</td>
</tr>
</tbody>
</table>
the signals from satellites 3 and 28. Clearly, the line-of-sight uncertainty increases (particularly for satellite 3), but the uncertainty along all six lines of sight remains well below the 15° threshold.

Accordingly, the local carrier replica for satellites 3 and 28 should be accurate when the signal returns to view, and the carrier phase discriminator should provide high-quality measurements of the phase error. Note that this analysis assumes that the phase discriminators from the channels tracking satellite 3 and 28 are ignored during the outage.

Based on the covariance analysis above, an outage is defined as a measurement update cycle during which fewer than four channels pass the phase lock test. Based on that criterion (i.e., four or more channels are phase-locked), the RTK-VPLL was in “vector” lock 98.7% of the time in the moderate-foliage environment and 90.3% of the time in the heavy-foliage environment. These percentages may be slightly optimistic because they assume that the carrier phase discriminators for the unlocked channels were successfully excluded from the navigation solution by the fault detection algorithms.

Additionally, the navigation solution accuracy during an outage depends on the motion of the receiver and the number of channels that are reporting high-quality carrier phase discriminators. For example, a non-accelerating or stationary receiver that is tracking three satellites may be able to maintain an accurate navigation solution for a few seconds before the error causes a loss of vector lock. This would only be possible if the filter had converged prior to the outage and the process noise matrix were tuned to near zero for the position and velocity states. Under these restrictions, the state covariance would be very low. The carrier phase residuals...
would map almost exclusively to the clock errors until the position and velocity state covariances increased to values comparable to the clock covariances. Let us consider the two plots in Figure 22. These two plots were constructed based on a covariance analysis of the growth in uncertainty during a measurement outage. In this case, only three measurements are used to update the navigation solution starting at the 20-s mark. At that point, the states of the filter are no longer fully observable. The plot on the left shows the simulation results obtained when using measurements from three satellites that result in a “good” dilution of precision (DOP), and the plot on the right shows the result of a “bad” DOP. Referring to Figure 20, the good-DOP satellites are 2, 19, and 28, and the bad-DOP satellites are 2, 6, and 19. In Figure 22, it is clear that satellite geometry is an important factor when navigating with fewer than four satellites. The navigator maintains a position solution that is accurate enough to maintain vector phase lock for more than 2 s with the better satellite geometry. When the poor geometry is used, the filter loses vector phase lock almost instantly.

7 CONCLUSIONS

A novel software receiver vector tracking architecture is described in this paper. The receiver combines a VDLL with a differential carrier phase-based VPLL that closes the phase tracking loop with RPV estimates, base station measurements, and fixed carrier ambiguities. The RTK-VPLL algorithm uses carrier phase measurements from the base station receiver and the states of the navigation filter to predict the phase of the received satellite signal and drive the carrier NCO. An RTK algorithm is used to estimate the carrier ambiguities off-line prior to initialization of the RTK-VPLL filter. The performance of the RTK-VPLL algorithm was evaluated via Monte Carlo simulations and experimentally collected GPS samples. Simulation analysis focused on the performance of the RTK-VPLL receiver in the presence of thermal noise. The RTK-VPLL algorithm was shown to maintain phase lock at carrier-to-noise ratios approximately 7 dB-Hz lower than a traditional scalar tracking PLL. Using data collected during static and dynamic tests in clear-sky environments, the accuracy of the RTK-VPLL navigation solutions was shown to be comparable to that of a survey-grade GPS receiver operating in RTK mode. The RTK-VPLL receiver was able to maintain phase lock in moderate-foliage environments where the reference receiver was not able to maintain a fixed-integer RTK solution. The need for a cycle slip correction algorithm in the software receiver was identified during testing in dense-foliage environments.
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